Journal of Applied Mechanics and Technical Physics, Vol. 43, No. 6, pp. 902–909, 2002

ON THE THEORY OF LOCAL ACOUSTIC PROBING OF BOREHOLE ZONES IN ROCKS

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UDC 532.529.534.2

A theory of local probing of borehole zones in porous and permeable rocks by means of acoustic waves is developed. Acoustic signals are assumed to propagate in an annular gap between the probe body and porous permeable wall of the borehole. Quantitative characteristics and special features of wave dynamics depending on the character of inhomogeneity of the porous medium are considered, in particular, in the case with radial fractures or a poorly permeable "crust" around the channel. The results obtained show that permeability and porosity of rocks in some cases exert a significant effect on evolution of acoustic signals in the borehole.

Introduction. The collector characteristics (permeability, porosity) of the bottomhole zone of oil and gas beds are improved using various physicochemical, hydrodynamic, and wave methods of bed treatment. One of the effective methods of real-time control of the state of the bottomhole zone before and after treatment seems to be the use of acoustic methods, which take into account the special features of signal dynamics at borehole sections with different permeability of the walls.

Acoustic methods are known to be used to inspect boreholes and rocks around them. Most methods of acoustic logging are based on interpretation of signals with the use of known features of propagation, attenuation, and reflection of longitudinal and transverse elastic waves in layered inhomogeneous fluid-saturated porous rocks. Some information on propagation of acoustic signals over the fluid in boreholes, in particular, washer fluid [1] are available in the literature. Nevertheless, we could not find reliable experimental data for systems with more or less known properties. This circumstance hinders the comparison of theoretical calculation results with experimental and field data. Thus, the main objective of the present work is the qualitative and quantitative analysis of the state of reservoirs in borehole zones on the dynamics of wave pulses propagating over the fluid in boreholes, depending on initial signal characteristics and probe parameters. The results of this study may be used for planning and conducting corresponding laboratory and field research.

In local acoustic control, it is assumed that the source and detectors of acoustic signals are located directly in the borehole sector under study.

Some aspects of wave dynamics in channels with permeable walls, which are filled by fluid, were considered in [2]. The main features of propagation and attenuation of acoustic waves in channels surrounded by a permeable space were studied in [3–6].

1. Propagation of Linear Waves on a Permeable Sector of the Borehole. We consider propagation of low-amplitude pressure waves in an annular gap. One side of the gap is a permeable wall of the borehole of radius a, and the other side is an impermeable wall (solid cylindrical body of the probe of radius a_1). The probe and the borehole have a common centerline. The probe surface contains a source D_1 and detectors of acoustic signals D_2, D_3, etc. The following assumptions are made: the channel is filled by the same acoustically compressible medium (liquid or gas) as the incompressible skeleton of the ambient porous state; the probe length L is significantly greater than the wavelength λ ($L \ll \lambda$), which, in turn, is greater than the gap between the probe body and the borehole ($\lambda \geqslant a - a_1$). In addition, the influence of viscosity [4] on momentum in the medium in this gap is neglected (signal evolution is mainly determined by effects of filtration to the ambient porous space).

Sterlitamak State Pedagogical Institute, Sterlitamak 453103. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 43, No. 6, pp. 142–150, November–December, 2002. Original article submitted December 8, 2000; revision submitted April 19, 2002.

0021-8944/02/4306-0902 $27.00

We write a system of linearized equations that describe propagation of perturbations between the surfaces of the probe body and the borehole surrounded by a homogeneous porous medium [3, 4]:

$$\frac{\partial \rho}{\partial t} + \rho_0 \frac{\partial w}{\partial z} = -\frac{2a\rho_0 u}{a^2 - a_1^2}, \quad \rho_0 \frac{\partial w}{\partial t} + \frac{\partial p}{\partial z} = 0, \quad p = C^2\rho, \quad a_1 < r < a; \tag{1.1}$$

$$m^{(1)} \frac{\partial \rho^{(1)}}{\partial t} + \frac{\rho_0}{r} \frac{\partial (ru^{(1)})}{\partial r} = 0, \quad \rho_0 \frac{\partial u^{(1)}}{\partial t} = -m^{(1)} \frac{\partial p^{(1)}}{\partial r} - \frac{m^{(1)}\mu}{k^{(1)}} u^{(1)}, \quad r > a. \tag{1.2}$$

Here p and ρ are the pressure and density perturbations, respectively, w is the velocity of the medium in the gap between the probe and cylindrical channel, u is the rate of filtration through the permeable walls of the channel, ρ_0 is the density of the undisturbed fluid, C is the velocity of sound in the fluid, $p^{(1)}$, $\rho^{(1)}$, and $u^{(1)}$ are the pressure, density, and filtration-rate distributions in the porous space around the channel, μ and $\nu = \mu/\rho_0$ are the dynamic and kinematic viscosity of the fluid, and $k^{(1)}$ and $m^{(1)}$ are the permeability and porosity of the space surrounding the channel.

Let, in the general case, the inner surface of the permeable sector of the borehole be covered by a thin ($\Delta a \ll a$) low-permeable annular "crust" with permeability k_c^*. Then, the intensity of fluid absorption through the permeable wall ($r = a$) is determined by the expression

$$u = u^{(1)}, \quad u^{(1)} = h(p - p^{(1)}) \quad [h = k_c^*/(\mu\Delta a)], \tag{1.3}$$

where h and Δa are the hydraulic resistance and thickness of the "crust" and $p^{(1)}$ is the pressure perturbation at the outer boundary of the low-permeable "crust."

If the channel is surrounded by an infinitely thick porous space, for propagation of acoustic perturbations in the gas (filtration processes in a porous medium near boreholes occur in layers whose characteristic length is significantly smaller than the thickness of the porous space around the borehole), system (1.1), (1.2) should be supplemented by the boundary condition

$$p^{(1)} = 0 \qquad (r \to \infty). \tag{1.4}$$

Neglecting hydraulic resistance of the "crust" ($h \to 0$), we can replace conditions (1.3) by

$$u^{(1)} = u, \qquad p^{(1)} = p \qquad (r = a). \tag{1.5}$$

From Eqs. (1.1), we can easily obtain

$$\frac{1}{C^2} \frac{\partial^2 p}{\partial t^2} = \frac{\partial^2 p}{\partial z^2} - \frac{2a\rho_0}{a^2 - a_1^2} \frac{\partial u}{\partial t} \qquad (a_1 < r < a). \tag{1.6}$$

We seek the solution of system (1.2)–(1.6) in the form of a travelling harmonic wave

$$\begin{gathered} u = A_u \exp{[i(Kz - \omega t)]}, \quad p = A_p \exp{[i(Kz - \omega t)]} \quad (a_1 < r < a), \\ u^{(1)} = A_u^{(1)}(r) \exp{[i(Kz - \omega t)]}, \quad p^{(1)} = A_p^{(1)}(r) \exp{[i(Kz - \omega t)]} \quad (r > a), \end{gathered} \tag{1.7}$$

where K and ω are the complex wavenumber and circular frequency of perturbations, respectively. Then, Eqs. (1.2) and conditions (1.3) yield

$$\begin{gathered} y^2 A_p^{(1)}(R) = \frac{1}{R} \frac{d}{dR}\Big(R \frac{dA_p^{(1)}(R)}{dR}\Big), \\ R > 1: \qquad A_u^{(1)} = -\frac{m^{(1)}k^{(1)}}{(-i\omega\rho_0 k^{(1)} + m^{(1)}\mu)a} \frac{dA_p^{(1)}(R)}{dR}, \\ R = 1: \qquad A_u^{(1)} = A_u, \quad A_u = h(A_p - A_p^{(1)}) \\ [y^2 = -i\omega a^2/\chi^{(1)} - \omega^2 a^2/C^2, \quad R = r/a, \quad \chi^{(1)} = k^{(1)}\rho_0 C^2/(\mu m^{(1)})]. \end{gathered} \tag{1.8}$$

Here $\chi^{(1)}$ is the piezoconductivity coefficient of the rock surrounding the borehole.

The general solution of Eqs. (1.8) has the form

$$A_p^{(1)}(R) = AI_0(yR) + BK_0(yR), \quad K_0(yR) = \int_0^\infty \exp{(-yR\cosh\,\xi)}\,d\xi, \quad I_0(yR) = J_0(iyR), \tag{1.9}$$

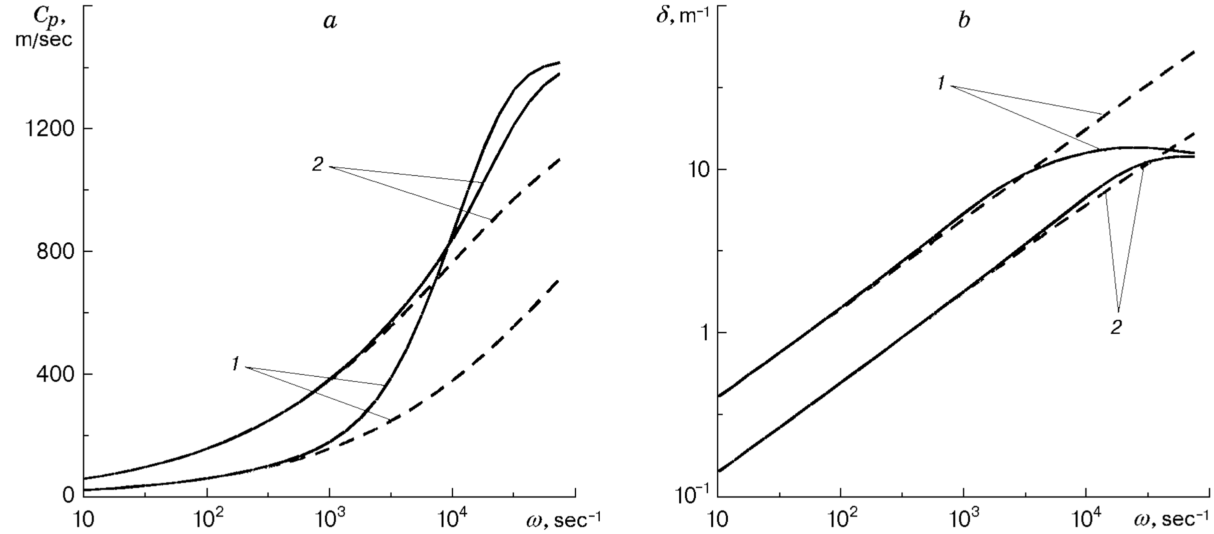


Fig. 1. Phase velocity (a) and attenuation factor (b) of perturbations as functions of frequency: the solid and dashed curves show the calculation results with and without allowance for inertial effects, respectively, for $k^{(1)} = 10^{-10}$ (curves 1) and 10^{-11} m^2 (curves 2).

where $K_0(yR)$ and $J_0(yR)$ are the zero-order Macdonald and Bessel functions [7], and A and B are arbitrary constants. For solution (1.9) to satisfy the boundary condition (1.4), we have to assume that $A = 0$. Then, on the basis of (1.8), we obtain

$$B = \frac{A_p}{K_0(y) - \gamma y K_0'(y)} \qquad \Big(\gamma = \frac{m^{(1)}k^{(1)}}{(-i\omega\rho_0 k^{(1)} + m^{(1)}\mu)ah}\Big). \tag{1.10}$$

With allowance for (1.9) and (1.10), the amplitude of filtration rate through the borehole walls is represented as

$$A_u = -\gamma h y A_p K_0'(y)/(K_0(y) - \gamma y K_0'(y)). \tag{1.11}$$

From the algebraic equation that follows from (1.6), with allowance for (1.11), we can obtain the dispersion expression

$$K = \frac{\omega}{C}\sqrt{1 - \frac{2a^2 m^{(1)}(\ln K_0(y))'}{y(a^2 - a_1^2)(1 - \gamma y(\ln K_0(y))')}}, \qquad K = k + i\delta, \quad C_p = \frac{\omega}{k}. \tag{1.12}$$

Here C_p and δ are the phase velocity and attenuation factor, respectively.

For short-wave perturbations, we cannot neglect inertia of the fluid during its filtration through porous permeable rocks around the borehole. An analysis of the expression for y in (1.8) shows that effects of inertia are significant in the range of frequencies that satisfy the condition $\omega \geqslant \omega_{(i)}$ $[\omega_{(i)} = C^2/\chi^{(1)}]$. In particular, for an ambient porous medium ($k^{(1)} = 10^{-11}$ m^2 and $m^{(1)} = 0.1$) saturated by water ($\nu = 1.06 \cdot 10^{-6}$ m^2/sec and $C = 1425$ m/sec), this frequency equals $\omega_{(i)} \approx 10^4$ sec^{-1}.

Based on the assumption $\lambda \geqslant a - a_1$ and taking into account that $\lambda \simeq 2\pi C/\omega$, we obtain the condition for the perturbation frequency $\omega \leqslant \omega_{(\lambda)}$ $[\omega_{(\lambda)} \approx 2\pi C/(a - a_1)]$. For instance, for a borehole filled by water with a gap $a - a_1 = 1$ cm, we obtain $\omega \approx 10^6$ sec^{-1}. Thus, within the high-frequency range, we can select the range $\omega_{(\lambda)} \geqslant \omega \geqslant \omega_{(i)}$, where inertial effects during fluid filtration to the ambient porous medium affect the evolution of waves in the gap between the probe body and the wall of the inspected sector of the borehole.

Figure 1 shows the phase velocity and attenuation factor of perturbations as functions of frequency in a borehole filled by water with and without allowance for inertial effects for $a = 5$ cm and $a_1 = 4$ cm.

Figure 2 shows similar dependences obtained in the presence (solid curves) and in the absence (dashed curves) of the low-permeable "crust" for $h = 10^{-11}$ m$^2\cdot$sec/kg, $k_c^* = 10^{-14}$ m^2, and $\Delta a = 0.5$ cm. A comparison of the solid and dashed curves shows that even a change in permeability of the main porous space around the channel by a factor of several dozens has only a weak effect on phase velocity and attenuation factor in the presence of the

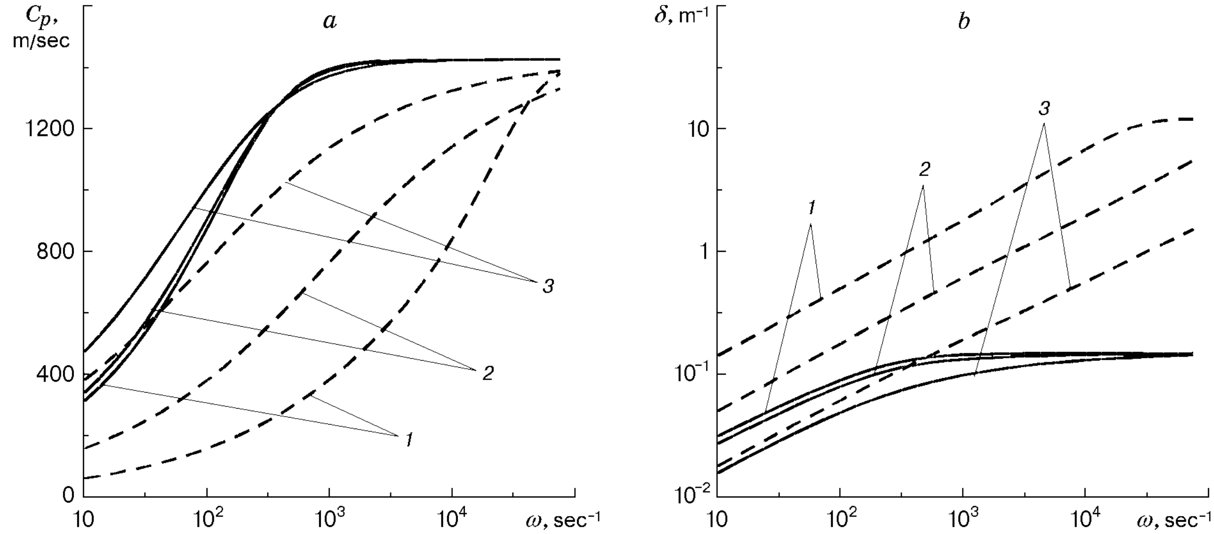


Fig. 2. Phase velocity (a) and attenuation factor (b) as functions of frequency: the solid and dashed curves refer to results obtained in the presence and absence of the low-permeable "crust," respectively, for $k^{(1)} = 10^{-11}$ (curves 1), 10^{-12} (curves 2), and 10^{-13} m^2 (curves 3).

"crust." Hence, in this method of probing, the action of a small low-permeable "crust" can completely screen the collector properties of the main porous space around the borehole.

For a rather wide range of frequencies that are of greatest interest for practice, we can neglect the term responsible for fluid inertia in the expression for y. Then, we obtain

$$|y| = \sqrt{\omega a^2/\chi^{(1)}} = \sqrt{\omega/\omega_\chi} \qquad (\omega_\chi = \chi^{(1)}/a^2).$$

Here ω_χ is the characteristic frequency at which the depth of penetration of filtration waves is of the order of the borehole radius [7]. In the range of high frequencies $|y| \gg 1$ ($\sqrt{\omega} \gg \sqrt{\omega_\chi}$), the dispersion equation (1.12) can be represented in the form

$$K = \frac{\omega}{C}\Big(1 + \frac{m^{(1)}}{\sqrt{2}\,[1-(a_1/a)^2]}\sqrt{\frac{\chi^{(1)}}{\omega a^2}}\,(1+i)\Big).$$

This equation, under the additional condition

$$\sqrt{\omega} \gg 2m^{(1)}\sqrt{\omega_\chi}/[1-(a_1/a)^2],$$

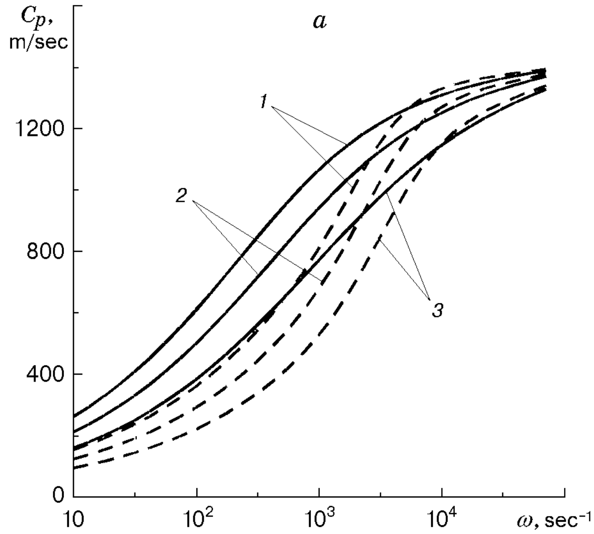
yields the following asymptotic dependences for phase velocity and attenuation factor:

$$C_p \simeq C, \qquad \delta = [a/(a^2 - a_1^2)]\sqrt{m^{(1)}k^{(1)}\omega/(2\nu)}\,.$$

A numerical analysis shows that the velocity of propagation of harmonic perturbations in boreholes surrounded by an ambient medium varies from zero ($C_p \ll C$) in the low-frequency range ($\omega \to 0$) to a value close to the velocity of sound in the medium ($C_p \simeq C$) in the high-frequency range. The determining physical factor that affects attenuation of perturbations is the kinematic viscosity of the fluid ν. The attenuation factor is inversely proportional to the kinematic viscosity of the fluid. In the case of a less viscous saturating fluid, attenuation of the wave in the gap is more significant. In addition, the attenuation factor in the range of both high and low frequencies increases with increasing permeability of the medium surrounding the channel and with decreasing gap size ($a - a_1$).

The solid curves in Fig. 3 show the phase velocity C_p and attenuation factor δ of acoustic perturbations as functions of frequency in the gap between the permeable wall of the borehole ($a = 5 \cdot 10^{-2}$ m) surrounded by a porous medium ($k^{(1)} = 10^{-12}$ m^2 and $m^{(1)} = 0.2$) and the probe body for different values of the probe radius. It follows from Fig. 3 that a decrease in the gap size (an increase in the probe radius a_1) leads to a significant decrease in phase velocity and to an increase in the attenuation factor.

The solid curves in Fig. 4 show the calculated oscillograms obtained by the fast Fourier transform with the use of the dispersion relation (1.12), which illustrate the evolution of a wave pulse with a characteristic duration



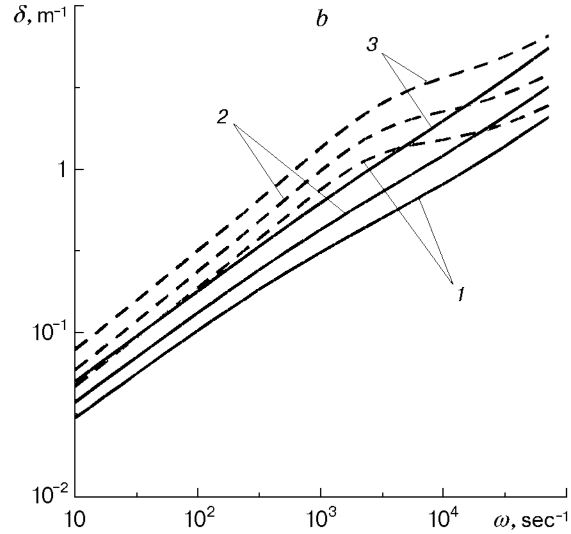


Fig. 3. Phase velocity (a) and attenuation factor (b) as functions of frequency: the solid and dashed curves refer to the absence and presence of radial fractures, respectively, for $a_1 = 0$ (curves 1), $3 \cdot 10^{-2}$ (curves 2), and $4 \cdot 10^{-2}$ m (curves 3).

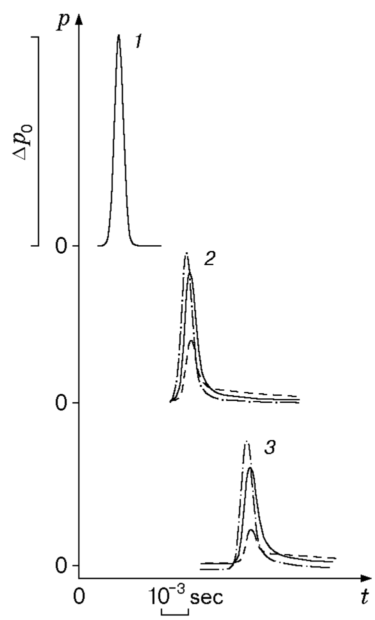


Fig. 4. Dynamics of the pressure pulse in the gap for $z = 0$ (1), 2 (2), and 3 m (3); the solid curves refer to the oscillograms obtained by the fast Fourier transform using Eq. (1.12) in the presence of the probe, the dot-and-dashed curves were obtained in the absence of the probe, and the dashed curves were obtained in the absence of the probe and in the presence of radial fractures.

$t_* = 10^{-3}$ sec in the gap with a probe radius $a_1 = 3 \cdot 10^{-2}$ m. Oscillogram 1 refers to the initial pulse ($z = 0$), and oscillograms 2 and 3 show the readings of gauges at a distance $z = 2$ and 3 m, respectively. In the case considered, we assumed that the outer boundary of the gap ($a = 5 \cdot 10^{-2}$ m) is a porous medium with parameters $k^{(1)} = 10^{-13}$ m^2 and $m^{(1)} = 0.2$. The dot-and-dashed curves refer to the case where the probe is absent or its radius is significantly smaller than the borehole radius ($a_1 \ll a$).

To obtain a more complete idea on signal evolution in the gap, the wave signal should have the maximum possible duration in time: the greater the signal duration, the greater depth of the porous medium around the borehole is reached by filtration perturbations. Thus, the signal evolution in the gap can offer more complete information about the state of porous beds.

2. Wave Dynamics in the Presence of Radial Fractures. We consider the evolution of waves in the case of a borehole surrounded by an inhomogeneous porous medium. Let the porous space around the borehole have n radial fractures (slotted channels with plane-parallel walls) filled by a more permeable porous medium than the main porous permeable medium. Such a situation occurs, for example, in the case of hydraulic breakdown of the bottomhole zone with subsequent filling of the fractures formed by the proppant. We assume that the fracture half-width b is significantly smaller than the borehole radius a ($b \ll a$). Then, we can write the equation of continuity, which generalizes Eq. (1.1), in the following form:

$$\frac{1}{C^2}\,\frac{\partial p}{\partial t} + \rho_0\,\frac{\partial w}{\partial z} = -\rho_0 u\,\frac{2(\pi a - nb)}{\pi(a^2 - a_1^2)} - \rho_0 \tilde{u}\,\frac{2nb}{\pi(a^2 - a_1^2)}. \tag{2.1}$$

Here $\tilde{u}$ is the rate of filtration of the fluid (liquid or gas) from the cylindrical channel to radial fractures. We consider an acoustic problem for such fractures. We introduce an additional coordinate axis in the radial direction along the fracture, which is counted off from the channel-wall surface ($r = a$). In describing acoustic waves in the fracture, we assume that the fracture is a channel with plane-parallel walls, which is surrounded by a porous medium of infinite thickness. The latter assumption means that the fractures are rather scarce ($n \ll \pi a/b$); therefore, the interaction of filtration fluxes around neighboring fractures is ignored. In addition, we assume that the wavelength in the fracture $\lambda^{(2)}$ is greater than the fracture height ($\lambda^{(2)} > b$).

Since the permeability of the porous medium in the fracture is greater than the permeability of the ambient space, the propagation of perturbations in the fracture is described by the following linearized system of equations [5]:

$$|x'| < b\colon \quad m^{(2)}\,\frac{\partial \rho^{(2)}}{\partial t} + \rho_0\,\frac{\partial u^{(2)}}{\partial x} = -\frac{\rho_0 v}{b}, \quad \rho_0\,\frac{\partial u^{(2)}}{\partial t} = -m^{(2)}\,\frac{\partial p^{(2)}}{\partial x} - m^{(2)}\,\frac{\mu}{k_c^{(2)}}\,u^{(2)}; \tag{2.2}$$

$$|x'| > b\colon \quad m^{(1)}\,\frac{\partial \rho'}{\partial t} + \rho_0\,\frac{\partial v'}{\partial x'} = 0, \quad v' = -\frac{k_c^{(1)}}{\mu}\,\frac{\partial p'}{\partial x'}; \tag{2.3}$$

$$|x'| = b\colon \quad v' = v, \quad p' = p^{(2)}. \tag{2.4}$$

Here x' is the value of the microcoordinate counted along the axis perpendicular to the upper (lower) wall of the fracture, the superscript $i = 2$ in brackets corresponds to parameters inside the fracture ($|x'| < b$), p', ρ', and v' are the pressure, density, and filtration-rate distributions in the porous space around the fracture, and $m^{(1)}$ and $k_c^{(1)}$ are the porosity and permeability of the main porous medium around the permeable sector of the borehole. The processes of wave propagation in the borehole and in the fractures are "matched" by the following boundary conditions on the borehole wall:

$$x = 0\colon \qquad u^{(2)} = \tilde{u}, \quad p^{(2)} = p. \tag{2.5}$$

With allowance for the equation of state from (1.1), we can obtain the following for Eqs. (2.2), (2.3):

$$|x'| < b\colon \qquad \frac{1}{C^2}\,\frac{\partial^2 p^{(2)}}{\partial t^2} + \frac{1}{\chi^{(2)}}\,\frac{\partial p^{(2)}}{\partial t} = \frac{\partial^2 p^{(2)}}{\partial x^2} - \frac{\rho_0}{b}\Big(\frac{\nu}{k_c^{(2)}}\,v + \frac{1}{m^{(2)}}\,\frac{\partial v}{\partial t}\Big); \tag{2.6}$$

$$|x'| > b\colon \qquad \frac{\partial p'}{\partial t} = \chi^{(1)}\,\frac{\partial^2 p'}{\partial x'^2} \qquad \Big(\chi^{(i)} = \frac{C^2 \rho_0 k_c^{(i)}}{m^{(i)} \mu}, \quad i = 1, 2\Big). \tag{2.7}$$

Based on Eqs. (2.6) and (2.7), with allowance for the boundary conditions (2.4), we can obtain a wave equation that describes the dynamics of linear perturbations in fractures filled by a porous medium and surrounded by a porous space. To represent the solution of Eq. (2.7) uniquely, we supplement the boundary conditions (2.4) by the initial conditions

$$p' = 0, \qquad |x'| > b, \qquad t = t_0.$$

According to Duhamel's principle [7], the solution of Eq. (2.7) has the form

$$p' = \int_{t_0}^{t} \frac{\partial U(x' - b, t - \tau)}{\partial t}\,p^{(2)}(x, \tau)\,d\tau. \tag{2.8}$$

Here $U(x', t)$ is the solution of the boundary problem that satisfies the condition $U(0, t) = 1$.

Equation (2.3), with allowance for (2.8) at $|x'| = b$, yields

$$v' = -\frac{k_c^{(1)}}{\mu}\frac{\partial p'}{\partial x'} = -\frac{k_c^{(1)}}{\mu}\frac{\partial}{\partial x'}\int\limits_{t_0}^{t}\frac{\partial U(x'-b,t-\tau)}{\partial t}\,p^{(2)}(x,\tau)\,d\tau$$

and

$$v = -\frac{k_c^{(1)}}{\mu}\Big(\frac{\partial p'}{\partial x'}\Big)_b = \frac{k_c^{(1)}}{\sqrt{\pi}\mu}\frac{\partial}{\partial t}\int\limits_{t_0}^{t}\frac{p^{(2)}(x,\tau)}{\sqrt{\chi^{(1)}(t-\tau)}}\,d\tau. \tag{2.9}$$

Substituting expression (2.9) into (2.6) and assuming, for generality, that $t_0 = -\infty$, we obtain the wave equation

$$\frac{1}{C^2}\frac{\partial^2 p^{(2)}}{\partial t^2} + \frac{1}{\chi^{(2)}}\frac{\partial p^{(2)}}{\partial t} = \frac{\partial^2 p^{(2)}}{\partial x^2}$$

$$-\frac{k_c^{(1)}}{b\sqrt{\pi}}\frac{\partial}{\partial t}\Bigg(\frac{1}{k_c^{(2)}}\int\limits_{-\infty}^{t}\frac{p^{(2)}(x,\tau)}{\sqrt{\chi^{(1)}(t-\tau)}}\,d\tau + \frac{1}{m^{(2)}\nu}\frac{\partial}{\partial t}\int\limits_{-\infty}^{t}\frac{p^{(2)}(x,\tau)}{\sqrt{\chi^{(1)}(t-\tau)}}\,d\tau\Bigg), \tag{2.10}$$

which describes the evolution of small perturbations in the fracture. The second term in the left side of this equation is determined by the force of viscous friction inside the fracture, and the second term in the right side is determined by filtration processes in the porous space around the fracture.

We seek the solution of Eq. (2.10) in the form of a travelling wave

$$p^{(2)} = A_p^{(2)}\exp\,[i(K^{(2)}x - \omega t)].$$

As a result, we obtain the dispersion equation

$$(K^{(2)})^2 = \frac{\omega^2}{C^2}\Big(1 + i\,\frac{\omega_c^{(2)}}{\omega}\Big)\Big(1 + \frac{m^{(1)}}{m^{(2)}y^{(2)}}\Big), \qquad y^{(2)} = \sqrt{-i\,\frac{\omega b^2}{\chi^{(1)}}} \tag{2.11}$$

$$[\omega_c^{(2)} = C^2/\chi^{(2)}, \quad \omega_\chi^{(2)} = \chi^{(1)}/b^2],$$

where $\omega_\chi^{(2)}$ is the characteristic frequency for which the depth of penetration of filtration waves is about half-height of the fracture.

According to (2.5), the amplitudes of pressure and filtration rate in the input cross section of the fracture ($x = 0$) should satisfy the conditions $A_p^{(2)} = A_p$ and $A_u^{(2)} = \tilde{A}_{\tilde{u}}$; therefore, based on the momentum equation in (2.1), we can write

$$\tilde{A}_{\tilde{u}} = m^{(2)}iK^{(2)}A_p/(\rho_0 i\omega - m^{(2)}\mu/k_c^{(2)}). \tag{2.12}$$

From the continuity equation (2.1), with allowance for (1.7), (2.11), and (2.12), we can obtain a dispersion expression similar to (1.12), which characterizes the dynamics of perturbations in boreholes with fractured-porous permeable walls:

$$K = \frac{\omega}{C}\sqrt{1 - 2m^{(1)}(\ln K_0(y))'y^{-1}\,\frac{(\pi a - nb)a}{\pi(a^2 - a_1^2)} + \frac{2nbm^{(2)}iC}{\pi(a^2 - a_1^2)\omega}\sqrt{\frac{1 + m^{(1)}/(m^{(2)}y^{(2)})}{1 + i\omega_c^{(2)}/\omega}}}.$$

Figure 3 shows the influence of the gap size between the borehole wall and probe body in the presence of radial fractures ($m^{(2)} = 0.2$, $b = 2\cdot 10^{-3}$ m, $k_c^{(2)} = 10^{-10}$ m^2, and $n = 4$) on the dependences of phase velocity and attenuation factor of acoustic perturbations on frequency (dashed curves). A comparison of the solid and dashed curves shows that the attenuation of acoustic perturbations is more pronounced in the presence of radial fractures, and the phase velocity of perturbations is smaller.

The dashed curves in Fig. 4 show the calculated oscillograms, which reveal the influence of fracturing on evolution of pulsed signals inside the borehole in the case the probe is absent (or its radius is significantly smaller than the borehole radius: $a_1 \ll a$). The calculations were performed for porous-medium parameters $a = 5\cdot 10^{-2}$ m, $k^{(1)} = 10^{-13}$ m^2, and $m^{(1)} = 0.2$ for the case of four radial fractures ($b = 2\cdot 10^{-3}$ m, $m^{(2)} = 0.2$, and $k_c^{(2)} = 10^{-10}$ m^2). The duration of the initial pulse was $t_* = 10^{-3}$ sec. A comparison of the dot-and-dashed (radial fractures are absent: $n = 0$) and dashed curves indicates that the presence of radial fractures significantly alters the scenario of signal evolution.

Conclusions. The results obtained show that the collector characteristics (permeability, porosity) of rocks around boreholes affect signal evolution. Hence, these results can be used to develop methods of control of collector characteristics of bottomhole zones in porous rocks.

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